

# THE MATHEMATICS OF ARISTOTLE'S THEORY OF TIME

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In many respects, Aristotle's theory of time found in the *Physics* IV.10-14 is simple. For Aristotle, time is nothing more than the measurement of one motion in respect to another motion. For practical purposes, Aristotle says time is generally the measurement of motion in respect to the heavenly bodies, whose motion he thinks is constant. (*Phy.* 223<sup>b</sup>12-24) In other ways though, Aristotle's theory is quite sophisticated. Although Aristotle conceives of time simply as the measurement of motion, Aristotle's accounts of measurement and motion themselves are fairly sophisticated, being built on a theory about infinite quantities that laid the groundwork for calculus.<sup>1</sup>

Topological properties of time both motivate and guide Aristotle's discussion.<sup>2</sup> There are two difficulties related to topological descriptions of time that Aristotle discusses while laying the background for his theory. The first difficulty concerns the relationship of the past, present, and future. The problem is how a series of presents, or 'nows', each having seemingly no duration, can make up time, which seems to have duration.<sup>3</sup> The second difficulty concerns the transition from one 'now' to the next. The problem is to provide a description of how one 'now' ends and becomes another 'now'.<sup>4</sup> (*Phy* 218<sup>a</sup>1-30) Topological properties of time also guide the development of his theory itself. Although much of his theory is developed from how time is actually measured, Aristotle often refers to the fact that time is not fast or slow, but rather defines fast and slow movements. (*Phy.* 219<sup>b</sup>1-5, 218<sup>b</sup>15) This observation seems to amount to the fact that time runs at a constant rate for an observer.

The claims of this paper are twofold. Although Aristotle never explicitly comes back to the problems given at 218<sup>a</sup>1-30 and gives a single solution, he does say that they must be worked out.<sup>5</sup> (*Phy.* 217<sup>b</sup>30) My first claim is that Aristotle does give a solution to the difficulties given at 218<sup>a</sup>1-30. I will argue that Aristotle solves these problems by reducing the topological 'now' of time to a unit of measurement, for which he already has a sophisticated theory. My second claim is that Aristotle's observation that time runs at a constant rate for an observer, a topological property, leads him to falsely infer that time cannot run at different rates for different observers, a metric property.<sup>6</sup> In effect, Aristotle's ambiguity between time's topological and metric properties, while leading him to a solution to several difficulties, causes him to miss an important implication for his relativistic theory of time.<sup>7</sup>

In this paper, I shall focus on the mathematical details of Aristotle's account of time and will not attempt to give a general interpretation or try to resolve all the difficulties of the text. My general interpretation of Aristotle's account of time will follow that of Annas (1975). The focus of my explication will be on the texts relevant to the connection between magnitude, motion and time and the derivation of time's continuity. It is in these texts that I believe the solution to the problems at 218<sup>a</sup>1-30 is to be found.

## THE PROBLEMS OF TIME AND THE DEFINITION

Aristotle begins his discussion of time just as he begins most of his discussions: by stating the topic, reviewing relevant contemporary views on the issue, and analyzing these views. He says:

“Next for discussion after the subjects mentioned is time. The best plan will be to begin by working out the difficulties connected with it, making use of the current arguments. First, does it belong to the class of things that exist or to that of things that do not exist? Then secondly, what is its nature?” (*Phy*, 217<sup>b</sup>30)<sup>8</sup>

The text for the difficulties to be considered is as follows.

Further, if a divisible thing is to exist, it is necessary that, when it exists, all or some of its parts must exist. But of time some parts have been, while others are going to be, and no part of it *is*, though it is divisible. For the ‘now’ is not a part: a part is a measure of the whole, which must be made up of parts. (*Phy*. 218<sup>a</sup>4-8)

Again, the ‘now’ which seems to bound the past and the future—does it always remain one and the same or is it always other and other? ... If it is always different and different ... the ‘nows’ too cannot be simultaneous with one another, but the prior ‘now’ must always have ceased to be. But the prior ‘now’ cannot have ceased to be in itself; yet it cannot have ceased to be in another ‘now’. For we may lay it down that one ‘now’ cannot be next to another, any more than a point to a point. (*Phy*. 218<sup>a</sup>11-19)

The difficulties then are to explain how time is made up of a series of ‘nows’, and how to coherently describe the transition from one ‘now’ to the next. Before Aristotle can solve these problems though, he needs a theory of time.

Aristotle’s theory of time is given by his definition of time. After discussing other definitions of time, Aristotle gives the following definition.

It is clear, then, that time is number of movement in respect of the before and after, that it is continuous since it is an attribute of what is continuous. (*Phy* 220<sup>a</sup>25)

Before discussing how Aristotle arrives at this definition or how it allows him to solve the above problems, several important points on his theories of number and movement must be explained.

## MAGNITUDE AND MOTION

Aristotle’s general view about numbers is the standard view held by the Greeks by the time of Euclid, and he gives his most detailed account of number in *Metaphysics* 10 and 13. What is important is that for Aristotle the concept of number was tied to the idea of measurement, or counting. For Aristotle, quantities are determined by defining a unit of measurement. Numbers then are something that are abstracted away from equinumerous quantities. This idea is summarized in the following two texts.

Measure is that by which quantity (*Here ‘poson’, or magnitude*) is known, and quantity qua quantity is known either by unity or by number, and all number is known by unity. Therefore all quantity qua quantity is known by unity, and that by which quantities are primarily known is absolute unity. Thus unity is the starting point of number qua number. (*Met*. 1052<sup>b</sup>19-24, italics mine)<sup>9</sup>

The general propositions in mathematics are not concerned with objects which exist separately apart from magnitudes and numbers; they are concerned with magnitudes and numbers, but not with them as possessing magnitude or being divisible. (*Met*. 1077<sup>b</sup>19-20)

Like his discussion about time, in these passages Aristotle is concerned with the reference of statements about numbers.<sup>10</sup> Aristotle's view is that statements about numbers are really statements about counting, and that counting itself is really a type of measuring. The view is a bit more complicated than this though, because measuring is always with respect to some arbitrary unit. (Annas 1975, 99)

For Aristotle, just as when one wants to measure the length of something he must first choose a unit of measure, such as the meter or kilogram, when one wants to measure quantities themselves he must choose a unit of measure. The unit of measure for quantities themselves though is simply "unity", or "one". Hence, for Aristotle, one is not a number, but merely the unit of measure for quantities in general.<sup>11</sup>

Aristotle broadly conceives of movement as being more general than just locomotion of macroscopic objects. What is important for this discussion is the connection between motion and infinite quantities. Aristotle gives his most detailed account of motion in *Physics* 3. There he summarizes this point.

Now motion is suppose to belong to the class of things which are continuous; and the infinite presents itself first in the continuous—that is how it comes about that the account of the infinite is often used in definition of the continuous; for what is infinitely divisible is continuous. (*Phy.* 200<sup>b</sup>15-20)

The fulfillment of what is potentially, as such, is motion—e.g. the fulfillment of what is alterable, as alterable, is alteration; of what is increasable and its opposite, decreasable, increase and decrease; of what can come to be and pass away, coming to be and passing away; of what can be carried along, locomotion. (*Phy.* 201<sup>a</sup>10-15)

While there are many controversies surrounding the interpretation of Aristotle's theory of motion—including problems translating the text—the general idea is that motion is the actualization of a potentiality. Specifically when it concerns locomotion, it is a change in place. A precise understanding though of Aristotle's concept of motion is not needed here. What is more important for this discussion is Aristotle's claim that motion is continuous, and hence that it is infinitely divisible.<sup>12</sup>

Aristotle's conception of infinite quantities, like almost all others before Cantor, was vague. The general idea is that quantities are infinite if they are "incapable of being gone through".<sup>13</sup> (*Phy.* 204<sup>a</sup>2) Another distinction Aristotle and most Greeks made was between infinite divisions and infinite additions. An Infinite quantity by addition is one that is formed by endless successive additions of the unit of measure, and an infinite quantity by division is a continuous quantity, one that can be endlessly divided. In the beginning of his discussion on the infinite and motion, Aristotle says:

Belief in the existence of the infinite comes mainly from five considerations: ... From the division of magnitudes—for the mathematicians also use the infinite (*Phy.* 203<sup>b</sup>15)

The problem, however, which specially belongs to the physicist is to investigate whether there is a sensible magnitude which is infinite ... Further, everything that is infinite may be so in respect of addition or division or both. (*Phy.* 204<sup>a</sup>1, 7)

Aristotle's conclusion is that all sensible magnitudes are infinite by division, but that no sensible magnitude is infinite by addition. That is, he thinks every sensible magnitude is continuous, but that no sensible magnitude is infinitely large.<sup>14</sup> (*Phy.* 206<sup>a</sup>15, 207<sup>b</sup>15)

The idea though that sensible magnitudes are continuous was the crux of Zeno's paradoxes, and if Aristotle was to hold the position and maintain that motion was possible, he would have to explain how an infinite series could be summed. Aristotle fails here, but his response is important none the less. He says,

In a way the infinite by addition is the same thing as the infinite by division. In a finite magnitude, the infinite by addition comes about in a way inverse to that of the other. For just as we see division going on *ad infinitum*, so we see addition being made in the same proportion to what is already marked off. For if we take a determinate part of a finite magnitude and add another part determined by the same ratio, we shall not traverse the given magnitude. But if we increase the ratio of the part, so as always to take in the same amount, we shall traverse the magnitude; for every finite magnitude is exhausted by means of any determinate quantity however small. (*Phy.* 206b2-12)

Of course, geometric series do converge and hence have finite sums. Aristotle's second claim though is more significant. Although Aristotle did not understand that a geometric series converged, he did understand that the sum of infinitesimal pieces had a non-zero magnitude, and had the idea of summing with arbitrarily small pieces.<sup>15</sup> Although this idea—that arbitrarily small pieces have non-zero magnitudes—was probably not original to Aristotle, it underlies Aristotle's entire mathematical description of magnitude, movement and time, and will allow him to derive the continuity of motion and time from his theory of number.

To summarize, Aristotle views numbers, or quantity, literally as magnitudes that have been measured by some arbitrary unit. In the case of the numbers themselves, they are the measure of general quantities and their unit is unity, or one. In the case of quantities of things, some other unit determines their magnitudes. One important property of motion is that its magnitude is continuous, it can be divided up into smaller and smaller units. Towards the end of his discussion of motion, Aristotle describes the relationship between magnitude, motion and time.

The infinite is not the same in magnitude and movement and time, in the sense of a single nature, but the posterior depends on the prior, e.g. movement is called infinite in virtue of the magnitude covered by the movement, and time because of the movement. (*Phy.* 207b21-25)

This derivative relationship between magnitude, movement and time will be important in understanding what Aristotle means by saying that time is the number of movement. (see also *Phy.* 219<sup>a</sup>10-14)

## TIME

Before advancing his own definition, Aristotle considers two contemporary accounts of time. (*Phy.* 218<sup>b</sup>1-20) The first view he considers is that time is the rotation of the heavens; the second that time is motion. The former view Aristotle dismisses on two grounds; first, that it is naïve, second that it cannot explain any of the problems associated with time. The latter view Aristotle dismisses because it leads to conclusions contrary to (what he takes to be) observed fact. First, if time were a motion or a kind of change, then time would be in the thing that changes or moves. However, Aristotle says that time is present everywhere, not only in the things that move. Second, motions and changes vary in speed, but Aristotle says time does not vary in speed and is always the same.

Despite the fact that time is not movement or change, Aristotle argues that time is not independent of movement and change. Aristotle's initial argument for this has been the subject of much debate and several interpretations. The text for the argument is as follows:

But neither does time exist without change; for when the state of our minds does not change at all, or we have not noticed its changing, we do not think that time has elapsed ... So, just as, if the 'now' were not different but one and the same, there would not have been time ... If, the, the non-realization of the existence of time happens to us when we do not distinguish any change, but the mind seems to stay in one indivisible state, and when we perceive and distinguish we say time has elapsed, evidently time is not independent of movement and change. (*Phy.* 218b20-31)

One common interpretation is that Aristotle is giving a naïve, phenomenological account of time—or at least basing this part of the argument on one. There are several arguments against this interpretation of the passage, the main argument being that this interpretation is incompatible with any reasonable interpretation of the rest of the rest of Aristotle's account. I will return to the passage once more detail has been filled in.

Aristotle next argues that time is connected to movement and change because any measurement of time requires marking off a "before" and "after" in motion—e.g., initial and final conditions.<sup>16</sup> (*Phy.* 219<sup>a</sup>25) Further, an initial and final condition can only be marked off if there is a third condition between the initial and final conditions to distinguish them. Aristotle then says that once we can distinguish an initial and final condition, we associate a 'now' with each, and call what is bounded between each 'now' time. In addition, Aristotle says,

When, therefore, we perceive the 'now' as one, and neither as before and after in a motion nor as the same element but in relation to a 'before' and an 'after', no time is thought to have elapsed, because there has been no motion either. On the other hand, when we do perceive a 'before' and an 'after', then we say that there is time. For time is just this—number of motion in respect of 'before' and 'after'. Hence time is not movement, but only movement in so far as it admits of enumeration. (*Phy.* 219<sup>a</sup>30-219b1)

It is difficult to see what Aristotle is saying here, but it is helpful to view the rest of the passage in the context of his ending comment about the enumeration of movement. If time is the enumeration of movement, then like his general account of numbers, time will involve arbitrary units and counting. The next point to remember is that by "motion" Aristotle does not strictly mean macroscopic locomotion, but merely the actualization of a potentiality.<sup>17</sup> Aristotle's view seems to be that the unit of time, the 'now', is determined by some motion relative to which it is not before or after. Time then is experienced when relative to the unit, the 'now', there is a before and after, e.g. the unit is able to measure some motion. Time then is the quantity of these units between some before and after. (For additional text, see *Phy.* 219<sup>b</sup>5-10, 220<sup>a</sup>15-25.)

Support for this interpretation of Aristotle's theory mainly comes from the general context. Annas in (1975) has strongly defended the interpretation.<sup>18</sup> There is additional textual support for it as well. While expanding more on his previous discussion, Aristotle says,

Time is a measure of motion and of being moved, and it measures the motion by determining a motion which will measure the whole motion, as the cubit does the length by determining an amount which will measure out the whole. (*Phy.* 221<sup>a</sup>1-5)

In passages like this one, Aristotle describes the process of picking an arbitrary unit, and then measuring another motion against the unit. As Annas points out, Aristotle uses the term "measure"

interchangeably with “number” throughout his discussion of time. (1975, 98) Other places where Aristotle compares time to number include *Phy.* 220<sup>a</sup>15-20, 220<sup>a</sup>25-30, 220<sup>b</sup>5-15, and 221<sup>a</sup>15-20.

One initial reaction to this account is to ask about the time elapsing during the unit. For example, if time is the enumeration of some unit, say like the minute, what about the time commonly assumed to be elapsing that is enumerated by a smaller unit, say like the second? This question poses no problem for Aristotle, whose theory entails that time can be enumerated in any way one likes, as long as there is a smaller unit available to measure the time. Further, his theory also entails that there will always be a smaller unit available to measure time because there will always be smaller movements. In turn, there will always be smaller movements because movement is continuous. This, in effect, seems to be the derivation Aristotle talks about at *Phy.* 207b21-25 (see quote p. 6). (Though Aristotle seems to waver on whether the continuity of movement is more fundamental than the continuity of magnitude, quotes like the above seem to suggest that the derivation goes back even further; that the continuity of movement is a result of the continuity of magnitude, which enumerates movement. Complicating this issue is Aristotle’s view of matter; Aristotle thinks that it is just a fact that matter is continuous—he rejects atomism.)

Another question to return to is Aristotle’s apparently phenomenological argument for the dependence of time on motion. This argument has been subject to some remarkably uncharitable interpretations, including Bostock (1980), Shoemaker (1969, 363), and Sorabji (1983, 74). Bostock says that Aristotle’s argument is that since time elapses only when we notice movement, time is dependent on moment—an obviously unsound and invalid argument. (1980, 148) Coope and Annas reject such an interpretation though for contextual reasons.<sup>19</sup> (Annas 1975, 101; Coope 2001, 359) Annas says that this argument is best understood in the context of the whole theory. She points out that since time is just an enumeration of motion, that time is dependent on someone to do the enumerating. If no one counts the time, then there is no time. This interpretation is supported by the text; Aristotle says as much in *Phy.* 223<sup>a</sup>22-28, where he returns to the topic while summarizing his own account.

With this in mind it is easy to see why Aristotle makes the argument that time is dependent on someone being there to notice movement or change—for him there is no time independent of an enumeration based on some arbitrary unit. This point is relevant for this discussion because it lends general support to Annas’ interpretation of the theory. It is clear that for Aristotle time is not something that we count or enumerate but rather *is* the counting or enumerating of motion with respect to another motion.<sup>20</sup>

Taking stock of what Aristotle’s theory achieves, Aristotle is able to derive the substantial result that time is continuous from the relatively weak assumption that time is merely a relative measurement. This derivation, along with an understanding of Aristotle’s account of time, rests on his account of the properties of motion and on his more general account of number and magnitudes. Aristotle’s physics program then in large part amounts to giving a theoretical account of number and magnitude and then defining motion and time and deriving results off the definitions.<sup>21</sup>

## SOLVING THE PROBLEMS

I have claimed above that the solution to the problems at 218<sup>a</sup>1-30 is to be found in Aristotle’s derivation of the continuity of time from the continuity of motion and magnitude. One difficulty though in finding any solution to the problems at 218<sup>a</sup>1-30 is that their solution requires some rigorous notion of ‘now’. As pointed out by Annas, it is not at all obvious that Aristotle has a single notion of ‘now’ in his theory of time, let alone a rigorous one. (1975, 109-111)

The problem, given by Annas, is that Aristotle seems to use the term ‘now’ to denote two different things.<sup>22</sup> In some passages Aristotle states that the ‘now’ is like a point dividing the

continuum of time into past and future, while in other passages Aristotle states that the ‘now’ is like a unit of time and hence that time is divided into a series of discrete ‘nows’. (*Phy.* 222<sup>a</sup>18-22) In some places, like *Phy.* 219<sup>b</sup>13-220<sup>a</sup>15, Aristotle seems to use the two ideas interchangeably. Annas summarizes the problem by saying, “Aristotle is prepared to compare the now with a unit, and this raises the same question as the first passage: how, without more explanation, are we to reconcile this with the repeated assertions that nows are not like tiny little periods, but are rather comparable to durationless points?”<sup>23</sup> (1975, 112)

Part of this confusion arises from Aristotle’s early discussion of the ‘now’, where he often draws an analogy between the ‘now’ of time and points on a line. Annas rightly points out that it is not clear from the text in just what ways the analogy is to hold. The key to understanding the analogy, to understanding how Aristotle can reconcile his use of the term ‘now’ as both a discrete unit and a durationless point, lies in the derivation of the continuity of time. A clue to this is found in *Phy.* 222<sup>a</sup>12, 222<sup>a</sup>18-22, where Aristotle says,

But this is not obvious as it is with the point, which is fixed. It divides *potentially* ... So the ‘now’ also is in one way a *potential* dividing of time, in another the termination of both parts, and their unity. And the dividing and the uniting are the same things in the same reference, but in essence they are not the same. (italics mine)

Two things seem clear in this passage. First, Aristotle does view his two notions of the ‘now’ as one and the same. Second, the analogy between the ‘now’ of time the point breaks down in that the ‘now’ only divides time *potentially*.

Why would Aristotle say the ‘now’ only divides time potentially? The derivation of the continuity of time makes this point clear: the ‘now’ only divides time potentially because the ‘now’ is the unit of time, and the unit of time is only potentially durationless (unlike the point of a line, which has no length). The unit of time is dependent on marking off some motion as the unit of time. Because motion is continuous, there is always a smaller and smaller motion that one could potentially mark off as the unit of time, but nonetheless the unit of time is only potentially durationless.

The solutions to the problems then are forthcoming if we take Aristotle as holding that the ‘now’ is an arbitrary unit of time that is potentially durationless.<sup>24</sup> The first problem of how time, something which has duration, can be made up of durationless ‘nows’, is solved by explaining that the ‘nows’ are only potentially durationless. In the context of the theory this solution is powerful because the background theory immediately provides an answer to the objection that the ‘nows’ of time seems to actually be durationless. If someone objects that the ‘nows’ of time seem to be actually durationless, Aristotle can answer that they seem to be actually durationless because there is always a potentially smaller unit of time. Nevertheless, the actual units of time with which one enumerates have duration.

The second problem of describing the transition from one ‘now’ to the next is solved by explaining that one ‘now’ succeeds the next in the same way that numbers succeed each other—the transition from one now to the next amounts to counting more units. The solution again is powerful because the background theory explains why the trouble arose in the first place—the ‘nows’ are mistaken as being actually durationless.

There are two immediate objections to this interpretation. First, one might object that the solutions do not work. Second, one might object that there is little textual support. In response to the first objection, it should be noted that this idea of potentially durationless ‘nows’ and potentially zero lengths is a crude version of the ideas behind both the infinitesimal and modern limit calculus. Unlike Annas and Waterlow—who see the idea of ‘nows’ as being both discrete and potentially durationless as contradictory—I do not see any problem here at all. In response to the second objection, *Phy.* 222<sup>a</sup>12, 222<sup>a</sup>18-22 makes it seem clear that Aristotle thinks both that

the two notions of 'now' are one and the same and that the 'now' is only potentially divides time. In light of the derivation of the continuity of time, the proposed solutions to the problems seem obvious.

### SIMULTANEITY AND A UNIVERSAL TIME

As is clear from Aristotle's view on the relationship between time and the act of enumerating, he does not think that there is time apart from our enumeration of motion. He does not seem to think that there is a universal flow of time. Nevertheless, Aristotle does explicitly hold that the enumeration of motion is independent of location. Aristotle's exact statement of his view is complicated and tied to his understanding of simultaneity.<sup>25</sup> The text is,

But other things as well may have been moved now, and there would be a number of each of the two movements. Is there another time, then, and will there be two equal times at once? Surely not. For a time that is both equal and simultaneous is one and the same time, and even those that are not simultaneous are one in kind; for if there were dogs, and horses, and seven of each, it would be the same number. So, too, movements that have simultaneous limits have the same time, yet the one may in fact be fast and the other not, and one may be locomotion and the other alteration; still the time of the two changes is the same if it is both equal and simultaneous; and for this reason, while the movements are different and separate, the time is everywhere the same, because the number of equal and simultaneous movements is everywhere one and the same. (*Phy.* 223<sup>b</sup>1-13)

Aristotle has two concerns in this text. First, he is answering the naïve objection that if time is merely the numeration of movement, would not different movements have times? He says that the enumeration of things does not depend on the type of thing (just designating a unit). Aristotle's second concern is more interesting. It seems that Aristotle wants to argue that "because the number of equal and simultaneous movements is everywhere one and the same", then events that begin and end at simultaneous 'nows' will have the same time.<sup>26</sup> The question though is why does Aristotle assume that the number of equal and simultaneous movements is everywhere one and the same? That is, why does Aristotle assume that the enumeration of movements is independent of position?

Aristotle's theory of time, as far as it is reconstructed here, does not suggest that the enumeration of movement is independent of position. In fact, Aristotle's theory of time necessitates that the enumeration of movement is dependent on many things, including the unit used to do the counting.<sup>27</sup> It is reasonable to assume that part of Aristotle's reason for holding this view comes down to what he sees as common sense. Since Aristotle thinks that the units used to enumerate movement are normally based on the movement of the heavens, and since he is concerned mainly with terrestrial movements, it probably seems obvious to him that the movements of the heavens—and hence the units of time—are constant throughout the earth.

Besides the fact that Aristotle probably held it as obvious that the enumeration of movement was independent of position, there is also textual evidence that he thought he had no reason to doubt this claim. While discussing the relevant phenomena connected with time, Aristotle says,

Again, change is always faster or slower, whereas time is not; for fast and slow are defined by time—fast is what moves much in a short time, slow what moves little in a long time, by being either a certain amount or a certain kind of it. (*Phy.* 218<sup>b</sup>15-19)

As Aristotle points out, experience suggests to us that time does not move faster or slower, but rather things move faster or slower in time. Aristotle recognizes that one of the qualitative properties of time is that it appears to run at a constant rate (See endnote 6). I suggest then that this topological property of time gives Aristotle confidence in his belief that the enumeration of movement is independent of location.

The problem is that this qualitative feature of time does not imply that the enumeration of time is always constant. Just because time appears to flow at a constant rate does not mean that time actually does flow at a constant rate. In fact, Aristotle's technical theory—that time is the enumeration of movement based on another movement—seems to leave this an open question. That Aristotle partially recognized the difference between the topological properties of time that he experienced and the metric properties of time that his theory was explaining is suggested by the following text.

It is clear, too, that time is not described as fast or slow, but as many or few and as long or short. For as continuous it is long or short and as a number many or few; but it is not fast or slow—any more than any number with which we count is fast or slow. (*Phy.* 220<sup>b</sup>1-5)

Even so, Aristotle seems to infer the wrong results from this distinction. Aristotle seems to think that because time is an enumeration that it will remain constant. Unlike numbers though, which enumerate based on a fixed unit, one or unity, time enumerates based on arbitrary units. More so, Aristotle holds that time is dependent on picking an arbitrary movement as a unit, but Aristotle himself holds that movements, at least in the case of locomotion, are relative to place or position. I suggest that Aristotle's theory of time might have suggested to Aristotle that, contrary to experience, the enumeration of time is *not* independent of position. Aristotle's theory seems highly suggestive anyway about the dependence of time on position.

That Aristotle misses this possible implication of his theory and still holds that the enumeration of movement is independent of position appears to be because Aristotle confuses the topological and metric properties of time. Aristotle, using his observation of largely topological features of time to guide the development of his theory, does not see some of the implications for the metric properties of time implied by his theory.

## SUMMERY AND FINAL CONSIDERATIONS

Ostensibly, I have argued that Aristotle does end up having a solution to the problems with which he begins his discussion of time and that he misses possible implications of his theory. Further, whether Aristotle intended to or not, one of the key parts to his solution is a reduction of time's topological properties to metric properties. Some, like Waterlow, have suggested that Aristotle's focus on time's metric properties limited the power and application of his theory. (1984, 113) I disagree though, and suggest that Aristotle was not limiting his account at all; rather he was trying to explain those topological properties that he opened his account with in terms of metrical properties.

There are many problems with Aristotle's theory of time and a suitably broad discussion of his theory is beyond the scope of this paper. Some, like Corish (1976) have argued that Aristotle's theory of movement and time is hopeless circular. Given though that many of the details of his account are still unclear, I would not want to make any claims on this issue. My goal rather has been to clear up some of these details, in particular the mathematical details of the theory.

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19. Waterlow, Sarah. 1984. "Aristotle's Now", *The Philosophical Quarterly*, 34.135, pp. 104-128

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<sup>1</sup> This claim obviously needs some qualification. While I will discuss the matter in more detail below, the general claim is that Aristotle's discussion of motion, magnitude, and infinite quantities—driven largely by his attempts to refute Zeno—later would provide an understanding of these things subtle enough for calculus. Though it is commonly thought that Leibniz and Newton developed calculus "from the ground up", historically there was much more continuity between their development of the calculus and medieval scholastic theories of motion and acceleration. For a discussion of some of this development, see Clagett (1948), Wallace (1968), (1969) and (1971).

<sup>2</sup> Here I follow Reichenbach in distinguishing between time's metric, or quantitative properties, and topological, for qualitative properties. (1956, pp. 19-20) This is a distinction Aristotle does not make.

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<sup>3</sup> Aristotle seems to be concerned about the same problem of existence Augustine discusses in *The Confessions*. If what is in the past no longer exists, and if what is in the future does not exist yet, how can anything exist, given that the present seems to be a durationless point, continually slipping from the past to the future. See *The Confessions*, Book XI, pp. 256-262 for Augustine's account of the problem.

<sup>4</sup> This problem is similar to the problems he considers with motion, where he is concerned with describing the transition from place to the next. The common problem with both moments in time and places in space is what happens when one moves from one moment or space to another.

<sup>5</sup> It should be pointed out that Aristotle does come close to giving a solution at *Phy.* 219<sup>b</sup>13-34, though this solution is far from clear. The problems with it will be addressed below.

<sup>6</sup> This claim may require clarification. I say that the appearance of time running at a constant rate for an observer must be a topological property because it would be impossible for an observer to measure a change in the rate of time without reference to another observer. Reichenbach discusses this problem in detail in (1927, 14-19). The problem is that any measurement of time requires the choosing of an arbitrary unit of measurement. The arbitrary unit itself though can only be assumed to remain constant, you could not detect a change in the arbitrary unit without comparing it to another arbitrary unit that was itself assumed to be consistent. Reichenbach gives the simple example of the meter stick. Once an arbitrary object is chosen as "the meter", measurement of an object proceeds by laying the stick down repeatedly across the object. The problem is though that the meter stick itself must be assumed to remain the same length as one picks it up and moves it along the object to be measured. Of course, measurements are often much more complicated than this, but the example suffices. In any case, Aristotle is basically right that time will run constant for an observer, but this is true because of how Aristotle defines units of time, and will hold true for any other definitional procedure.

<sup>7</sup> Of course, I do not mean for my terminology to suggest a connection between Aristotle's theory of time and Einstein's. There is no connection. Aristotle's theory though is a relativistic theory in that for him, time is a measured quantity relative to some motion.

<sup>8</sup> All translations from *The Complete Works of Aristotle*. Following normal conventions, when citing Aristotle I will cite the line numbers, which are standard in all translations, instead of citing page numbers.

<sup>9</sup> When Aristotle speaks of "number qua number", or "quantity qua quantity", he is referring to an abstractive process of considering number or quantity apart from extraneous information. That is, "number qua number" refers to the essential properties of numbers in general, as opposed to properties of particular numbers. The terminology and the phrase come from the general method Aristotle believes is involved in such abstract considerations. One selectively focuses on the essential features of numbers, insofar as they are numbers, apart from anything else. Likewise for quantities, or any other "qua" operator.

<sup>10</sup> Annas concurs, and says that these two passages are the only places where Aristotle gives a clear account of his view. Annas 1975, p. 99

<sup>11</sup> There are many reasons to reject Aristotle's account of number, but here we only need to understand it in order to understand his definition of time. For a much more detailed account of Aristotle's philosophy of mathematics that concentrates on the abstractive nature of his account, see Lear (1982).

<sup>12</sup> To say the least, I have hand-waved through this difficult and complex part of Aristotle. Treatments and interpretations of Aristotle's account of motion vary widely though once one gets beyond the textbooks. Further, although of course Aristotle's discussion of the *causes of motion* turn out to be disastrously wrong and, as noted, his definition of motion might even be incoherent, his mathematical description of motion of continuous is ingenious.

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<sup>13</sup> Aristotle actually discusses different ways the term “infinite” is used. This though is the central idea.

<sup>14</sup> Aristotle has some rather elaborate arguments for this point, almost all of them being theoretical arguments turning on his definitions of position and place. A “sensible magnitude” seems to just be any type of physical measurement, be it the measurement of an object or of a distance.

<sup>15</sup> To Aristotle’s and Zeno’s credit, there is more to this paradox than my summery suggests. In addition, Aristotle’s math may not be wrong, depending on what he means by “same ratio”. In (1967), Apostol describes the situation thus. If Aristotle is indeed talking about a geometric series like  $\sum_{n=0}^{\infty} (1/2^n)$ , then he is incorrect and the series will converge (will sum to a finite number). However, this interpretation is not strictly what Aristotle says, for what the math really says is that there exists a limit to the sequence of the partial sums of this form. Of course, one might argue that this is the only possible mathematical interpretation to what Aristotle says. Further, Apostol points out that if we tweak our interpretation of the paradox slightly to involve the series  $\sum_{n=0}^{\infty} (1/n)$ , this series does not converge and will not sum to a finite number. (pp. 375-7)

<sup>16</sup> One may equate “before” and “after” in motion instead with initial and final positions, but this would not be strictly correct. Aristotle conceives of motion much more generally than macroscopic locomotion. Also, it should be noted that some scholars, such as Bostock (1980) have found extreme difficulty with understanding the reference of the terms “before” and “after”. Bostock worries whether the “before” and “after” reference “before” and “after” in place, in movement, or in time. He says that the standard view from Aquinas is that it is a “before” and “after” in movement, but argues that both Aquinas’ view and the view that it is a “before” and “after” in place are incoherent. Instead Bostock argues that Aristotle must mean a “before” and “after” in time, but points out the circularity in this. (1980, p. 151) There are several points that should be made. First, Bostock, while recognizing that Aristotle has a more general idea of motion than locomotion, confuses his arguments by assuming here that Aristotle only thinks of movement as locomotion. Second, Bostock also seems to assume that Aristotle is only working with one conception of ‘now’, which is untrue. Finally, Bostock seems content here to argue over this passage in isolation, and to appeal to his own senses about what Aristotle must mean. My approach, on the other hand, is to give Aristotle the fairest read, using vocabulary that accords with his, and then to sort out the details of the passage *in context*. My suggestion to take the “before” and “after” as initial and final conditions may seem to also beg the question of whether they are initial and final conditions in place or in movement, but this question is irrelevant in the context of Aristotle’s theory—movement is a derivative quantity of condition. Support of this suggestion comes from Aristotle himself, who explicitly says that the “before” and “after” in place, movement, and time are derivative on each other. See *Phy.* 219a15-20. I think Bostock is here building a straw-man at best.

<sup>17</sup> A further difficulty is that Aristotle uses the term ‘now’ in two ways, first as a reference to the unit of time, and second as a reference to the divide between the past and the future.

<sup>18</sup> This is probably the first place in the modern literature where this interpretation was fully developed.

<sup>19</sup> Although I will not discuss her argument in detail here, Coope’s basic argument is that this phenomenological interpretation of Aristotle’s argument contradicts what Aristotle says elsewhere about the topic, and that the argument, on such an interpretation, would be remarkably unlike Aristotle. She claims that Aristotle is not attempting to give here a deductive argument for the claim, but rather simply following his normal method of starting from common notions about a topic. So here Aristotle is then merely elaborating on the common belief that time required motion, and finding no reason to reject the claim, accepts it.

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<sup>20</sup> Newton would say that Aristotle has done violence to the Scriptures, but then again Einstein committed sacrilege himself. Newton explains in the Scholium of the Definitions section of *The Principia* that there is an “Absolute, true, and mathematical time, in an of itself and of its own nature, without reference to anything external, (which) flows uniformly and by another name is called duration”, as opposed to “Relative, apparent, and common time (which) is any sensible and external measure of duration by means of motion”. (p. 408, 414) Aristotle of course would have simply rejected this distinction, though his reasons for doing so aren’t really correct.

<sup>21</sup> Of course, the definitions are motivated by observation, or at least intuition. This Physics program though is not unusual for Aristotle; it merely follows his general method in science.

<sup>22</sup> Actually, Aristotle speaks of the ‘now’ in at least three ways, but the additional ways he speaks about the now besides as the unit of time and as the divider between the past and future are generally popular ways of speaking about the now. In these passages Aristotle is concerned with explaining what people mean when they use the term now in conversation, not with any legitimate use of the term.

<sup>23</sup> Waterlow, in (1984, 104) agrees that this is a problem. While I have focused on Annas’ discussion of this topic, Waterlow also provides a fairly detailed reconstruction of Aristotle’s view of the ‘now’. Waterlow’s discussion is also partly motivated by Aristotle’s apparently contradictory claims that the ‘now’ is both like a discrete unit and a durationless point separating the future from the past. Waterlow’s solution though is quite different than mine. She reconstructs Aristotle’s view of the ‘now’ as terminal events. While Waterlow’s reconstruction seems to achieve some success, it’s primary downfall seems to be that it does not account for the importance Aristotle places on arbitrary units. Instead, Waterlow focuses on Aristotle’s discussion of simultaneous events. See Waterlow (1984, 108-113).

<sup>24</sup> This of course is one interpretative advantage of my account over Waterlow’s. My interpretation of the ‘now’ as the unit of time allows for an elegant solution to the problems that motivate Aristotle’s discussion of time.

<sup>25</sup> Aristotle also provides a less detailed account of this at *Phy.* 220<sup>b</sup>6-14.

<sup>26</sup> Some might wonder here whether Aristotle’s use of the term ‘simultaneous’ is circular or not. I do not think it is. Aristotle seems to have the idea that simultaneous events are events that take place in the same ‘now’. Since a ‘now’ is just an arbitrary unit, say like the motion of the sun across the sky, it is easy to see whether two motions begin or end inside the same unit of time without reference to time itself, e.g., without enumerating anything already. It’s just an empirical observation.

<sup>27</sup> It is also dependent on the motion, which is dependent on the *position*, etc...